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DEPARTMENT OF THE NAVY BUREAU OF SHIPS

RESEARCH AND DEVELOPMENT REPORT
on
THE SHEAR BUCKLING PROPERTIES
of
SQUARE PANELS OF WOVEN ROVING LAMINATE

Lab. Project 5834-2, Final Report

SR 007-03-04, Task 1009

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SUMMARY

Experimental verification has been made of the previously developed theoretical analysis, and corresponding design data sheet, which permits the prediction of critical shear buckling stresses for square panels of orthotropic material with clamped edges for any angle ω that the principal axis of elasticity makes with the reference edge. The theory correctly predicts low values of buckling stress for $\omega = 135$ degrees for materials in which the elastic constants in the two principal directions are widely different. Experiments with a woven roving laminate, in which the ratio of moduli in the principal directions is 2:1, show buckling stresses 15 percent lower than the predicted values. The differences are accounted for by the knowledge that theory predictions are about 5 percent high and by variations in properties and dimensions of actual materials which preclude the development of full buckling strength. A recommendation is made to utilize the data sheet with due consideration given to these differences.

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ADMINISTRATIVE INFORMATION

Ref: (a) BUSHIPS ltr R007-03-04 Ser 634C4-662 of 11 Jul 1961
(b) MATLAB NAVSHIPYDNYK Project 5834-1 Final Report of 15 Nov 1960
(c) MATLAB NAVSHIPYDNYK Project 5834-1, Progress Report 1 of 26 Apr 1960
(d) MATLAB NAVSHIPYDNYK Project 5834-1 Progress Report 2 of 19 Sep 1960
(e) MATLAB NAVSHIPYDNYK Project 5834 Final Report of 26 Dec 1956

1. Reference (a) authorized the Material Laboratory to conduct an experimental investigation on the shear buckling of square panels using a laminate reinforced with woven roving. The purpose of these experiments was to obtain data which could be used to verify the design data sheet developed and presented as Figure 7 of reference (b) and reproduced as Figure 1 of this report. The design data sheet represents the final results of a theoretical analysis leading to the approximate solution of the buckling problem of a square panel of orthotropic material with clamped edges under shear loading for any angle ω that the principal axis of elasticity makes with the edge of the panel. Details of the analytical work were reported under reference (c). It was shown, that the buckling load for an orthotropic material under shear loading varies with the angle and that the buckling load is very low at $\omega = 135°$ when the elastic constants in the two principal directions are widely different, as indicated by low values of D_2/D_1 on Figure 1. A laminate reinforced with woven roving, having elastic constants in the principal directions in the ratio of approximately 2 to 1, was used to verify the above theory.

ACKNOWLEDGEMENT

2. The assistance of the following Material Laboratory personnel is acknowledged: P. Abramov and L. Baker in obtaining and assembling experimental data; R. Winans and N. Fried in the fabrication of the woven roving laminate and consultative service on reinforced plastics.

The Bureau of Ships Program Manager for the work reported herein is E.A. Bukzin, Code 342A, and the Bureau of Ships cognizant engineer is W.R. Graner, Code 634C3.

OBJECT

3. The object of this work was to confirm by experiment the validity of the design data sheet for calculating the buckling stresses for glass reinforced plastic laminate square panels in which the elastic constants in the two principal directions are widely different.

DESCRIPTION

4. The woven roving laminate used in this work was fabricated in the Material Laboratory in the form of sheets 46 inches by 50 inches from 5 plies of style 205-50 glass cloth, in a parallel layup, and Polylite 8000 polyester resin with 2 percent ATC paste hardener added. The laminate was press cured 30 minutes at 150°F, 30 minutes at 175°F and 2 hours between 225 and 250°F to 1/8 in. stops. Two sheets were made in this manner and are referred to as sheet 1 and sheet 2 in this report.

METHOD

5. The mechanical properties of the woven roving laminate were determined from sheet 1. Specimens were cut at angles of 0, 45, 67.5, 90, 112.5 and 135 degrees to the direction of the warp. Three specimens were used to determine

the tensile, compressive, and flexural properties in each direction. Strain gage techniques, using procedures described in detail in reference (d), were employed for determining the modulus of elasticity in tension and Poisson's ratio.

6. Panel shear properties were determined with the shear loading jig described in reference (b). The jig permits clamping a specimen of the laminate between two heavy frames which provide for a panel shear test on a square 8 inches by 8 inches panel with clamped edges. Tests were conducted on two series of specimens, one series from sheet 1 and one series from sheet 2 of the woven roving laminate. Each series of specimens were cut so as to permit shear loading the panels with the warp of the laminate making angles of 0, 45, 67.5, 90, 112.5 and 135 degrees with the reference edge. Dial gages were mounted on the jig frames in a manner to permit measurement of shear strain in the panel and lateral deflection at the center. The load was increased in increments of 100 pounds and corresponding dial gage readings were recorded up to and beyond the buckling load. The time for applying and holding each increment of load was approximately 30 seconds.

RESULTS

7. The mechanical properties of the laminate used in this work are summarized in Table 1. Each value tabulated represents an average of 3 results. Considerable variation of mechanical properties with loading direction is quite evident as would be expected for orthotropic materials. The tensile moduli, and ultimate strengths, in the principal directions, $\omega = 0$ and $\omega = 90$, are seen to be in the ratio of 2.1 and 2.5, respectively. Corresponding ratios of elastic moduli in compression and flexure are calculated to be 1.8 and 2.2, respectively.

8. Shear stress-strain curves, plotted from data obtained from the panel shear tests, are given in Figures 2 to 4. Figure 2 is a typical curve from which the modulus of elasticity in shear was determined. Figures 3 and 4 are shear stress-strain curves for various angles ω that the principal axis of elasticity, warp, makes with the reference edge of the panel. Values of the modulus of elasticity in shear, G, for sheets 1 and 2 of the laminate investigated, are given in Table 2 for various angles ω .

9. Figures 5 and 6 are curves of shear stress versus lateral deflection at the panel center as plotted from data obtained during the panel shear tests. In Figure 5 the curve for $\omega = 0$ is high in comparison with the curve for $\omega = 90$ degrees in Figure 5 and $\omega = 0$ and $\omega = 90$ degrees in Figure 6. The reason for this is the greater thickness for this panel and when correction is made for thickness the curve falls in proper relative position.

ANALYSIS

10. For orthotropic materials the equations which relate the elastic moduli for any direction that the principal axis of elasticity makes with the stress direction are as follows:

$$\frac{1}{E'_{11}} = \frac{\cos^4\omega}{E_{11}} + \frac{\sin^4\omega}{E_{22}} + \left[\frac{1}{G} - \frac{2\nu_{12}}{E_{11}}\right]\sin^2\omega\cos^2\omega \qquad (1)$$

$$\frac{1}{G'} = 4\left(\frac{1}{E_{11}} + \frac{1}{E_{22}} + \frac{2\nu_{11}}{E_{11}}\right)\sin^2\omega\cos^2\omega + \frac{1}{G}\left(\cos^2\omega - \sin^2\omega\right)^2 \qquad (2)$$

$$\frac{\nu'_{11}}{E'_{11}} = \frac{\nu_{11}}{E_{11}}\left(\cos^4\omega + \sin^4\omega\right) + \left(\frac{1}{G} - \frac{1}{E_{11}} - \frac{1}{E_{22}}\right)\sin^2\omega\cos^2\omega \qquad (3)$$

where E_{11}, E_{22}, G and ν_{11} are the elastic constants corresponding to the principal directions, and E'_{11}, E'_{22}, G' and ν'_{11} are the elastic constants corresponding to the loading direction which makes an angle ω with the principal directions.

11. The elastic constants given in Tables 1 and 2 correspond to the primed terms of the above equations. Substitution of these data into equations (1), (2) and (3) above and application of the method of least squares yield the following principal elastic constants for the woven roving laminate investigated.

$$E_{11} = 4.064 \times 10^6$$
$$E_{22} = 2.112 \times 10^6$$
$$\nu_{11} = 0.180$$
$$G = 0.552 \times 10^6$$

Substitution of these constants back to equations (1), (2) and (3) gives the equations which permit the calculation of the elastic constants for any loading direction. These equations are shown plotted on Figure 7. Actual test data, which are shown plotted with these curves for comparison, are seen to fall fairly close to the curves and therefore, the woven roving laminate investigated is considered to behave very much like an orthotropic material. In those instances where the test data do not fall on the curves, the differences are probably due to variation in properties from point to point in the laminate resulting from fabrication variables.

12. The results of the panel shear tests on the two sheets of woven roving laminate covered by this report permits a verification of the design data sheet, Figure 1 for the case where $D_2/D_1 = 0.52$. Referring to the buckling results plotted on Figures 5 and 6, it will be observed that the curves show a slowly increasing lateral deflection until a "knee" is reached at which point the deflection increases rapidly. In the ideal case the lateral deflection would be zero until the buckling stress is reached at which point the lateral deflection would start suddenly and continue with very little increase in stress. The gradual increase in lateral deflection at loads below the buckling stress may be attributed to variations in properties and dimensions of the panels and when these variations are large, the lateral deflections at low loads may be expected to increase rapidly. This behavior makes it difficult to determine the exact buckling stress from experimental data. However, the results reported in reference (e), on aluminum panels, for which exact buckling stresses could be calculated, showed that the point of inflection on the shear stress versus lateral deflection curve may be taken as a fair indication of the buckling stress. The application of this procedure to the buckling curves of Figures 5 and 6 reveals that considerable freedom is possible in determining the

buckling stresses from the points of inflection. Further study of the buckling curves shows that the more pronounced points of inflection tend to fall at a lateral deflection of approximately 0.050 inches. In order to make the determination more objective the buckling stresses were taken from the curves at those points where the lateral deflection is 0.050 inches. A review of the curves will show that these points could very well be taken as the points of inflection and therefore the procedure tends to make the selection of the buckling stresses unbiased.

13. Calculated values of the critical buckling shear stresses for the panels on which experiments were conducted may be determined with the aid of the design data sheet, Figure 1. In this connection it will be observed from the design data sheet that the modulus of elasticity in shear, G, is required for the calculation of D_3/D_1. However, Figures 2, 3 and 4 show that the shear stress-strain curves for ω =0 and ω =90 degrees are not linear up to the values of critical stress indicated by the buckling curves, Figures 5 and 6, and therefore the tangent modulus of elasticity corresponding to the buckling stresses should be used in the calculations. The curve of tangent modulus versus shear stress shown on Figure 8 was developed for this purpose from the shear stress-strain curve given in Figure 2. The tangent modulus curve is shown broken for stresses above the buckling stress. In making the calculation of the critical buckling shear stress a reiteration method was used because the tangent shear modulus of elasticity at the unknown critical stress was needed. A first approximation was made using the initial tangent shear modulus which yielded an approximate critical shear stress. By selecting the tangent modulus at this stress from Figure 8 a second approximation of the critical shear stress was made. A repetition of this procedure gave a third approximation of the critical shear stress which was sufficiently close to the second approximation to be considered acceptable. Although it is customary to use the flexural moduli of elasticity in the calculation of buckling stresses, the values obtained from the tension tests were used in the calculations for this report because it was considered that these values were more reliable. In addition, the individual tensile moduli E_{11} and E_{22} were found to be constant up to and beyond the critical stresses and as a result the corresponding tangent moduli were not needed for the calculations.

14. Experimental values of critical buckling shear stresses and corresponding calculated values determined with the aid of the design data sheet, Figure 1, are given in Table 3. The experimental values have been corrected for thickness to correspond to the calculated values for a panel thickness of 0.1 inch. The ratio of experimental to calculated values vary between 0.84 and 1.16. A better indication of how the experimental and calculated values compare over the range of angles ω is obtained from the plotted results shown in Figure 9. The solid curve is similar to those shown on the design data sheet but was specifically calculated to correspond to the properties of the woven roving laminate with E_{22}/E_{11} equal to 0.52. The experimental data are seen to follow the general trend of the curve, thus confirming the theory which formed the basis for the curve. Although there is some scatter in the experimental data, there is no doubt that the critical buckling shear stresses at ω =135 degrees for this type of material will be low as predicted by the developed theory and design data sheet. Except for the values at 45 and 67.5 degrees, the experimental buckling stresses are about 15 percent lower than those predicted by theory. It is known from the analysis given in reference (b) that the design data sheet predicts buckling stresses that are about 5 percent high. On this basis the experimental values are about 10 percent too low. These results are considered good when due

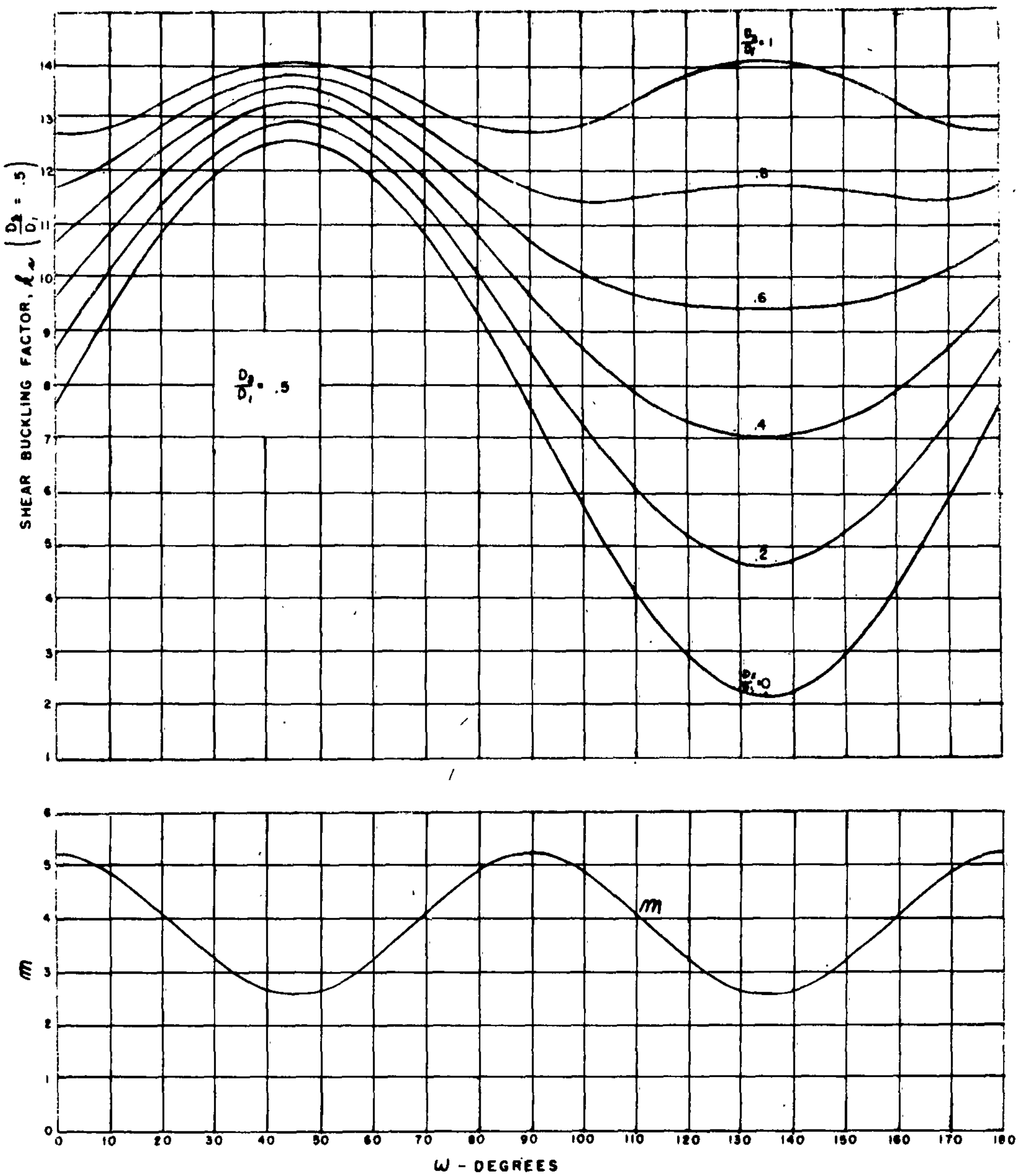
consideration is given to the type of test conducted and variability in thickness and properties of the laminate used in the investigation. The relatively high values in the neighborhood of ω =45 degrees are not readily explained. Similar results were obtained in the work reported in reference (b). It appears possible that since the material is not homogeneous, it may not behave as a perfectly orthotropic material and may show small variations from theoretical behavior by giving high values of buckling stress in the neighborhood of ω =45 degrees and low values for all other angles.

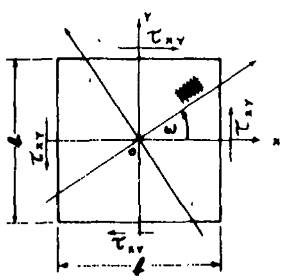
CONCLUSIONS

15. The design data sheet, Figure 1, has been confirmed and is suitable for predicting the critical buckling shear stresses of square panels with clamped edges for any angle that the principal axis of elasticity makes with the reference edge.

16. Buckling experiments on reinforced plastics give, in general, critical stresses which are about 85 percent of the values predicted by the design data sheet. These low values may be attributable, in part, to the variations in properties and dimensions of real materials which detract from the perfection required for the development of full strength in exacting buckling experiments.

17. The design data sheet may be used in practice provided the factor of safety gives due consideration to the predictions that are 15 percent high and to imperfections in real materials which preclude the development of full buckling strength.





$$\tau_{cr} = k_\omega \frac{\pi^2 D_1}{\ell^2 t}$$

$$k_\omega = \left(k_\omega\right)_{\frac{D_3}{D_1}=.5} - m\left(.5 - \frac{D_3}{D_1}\right)$$

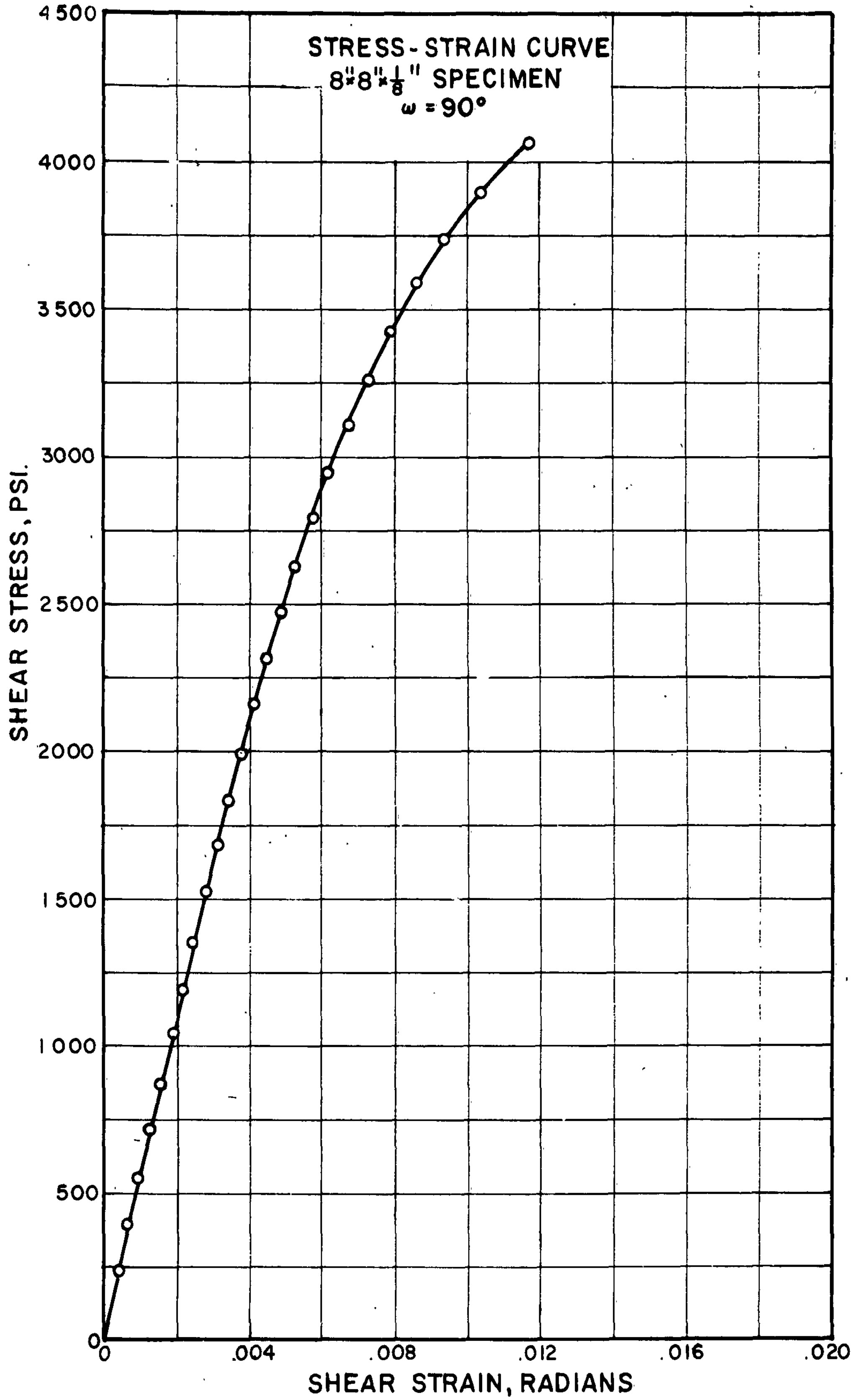
$$D_1 = \frac{E_{11} t^3}{12(1-\nu_1 \nu_{22})}; \quad D_2 = \frac{E_{22} t^3}{12(1-\nu_1 \nu_{22})}; \quad \frac{D_2}{D_1} = \frac{E_{22}}{E_{11}}$$

$$D_3 = \nu_1 D_2 + \frac{2Gt^3}{12}; \quad \frac{D_3}{D_1} = \nu_1 \frac{E_{22}}{E_{11}} + 2\left(1 - \nu_1 \nu_{22}\right)\frac{G}{E_{11}}$$

SHEAR BUCKLING FACTORS, k_ω, FOR ORTHOTROPIC SQUARE PLATE WITH CLAMPED EDGES FOR VARIOUS ANGLES, ω, OF PRINCIPAL AXIS

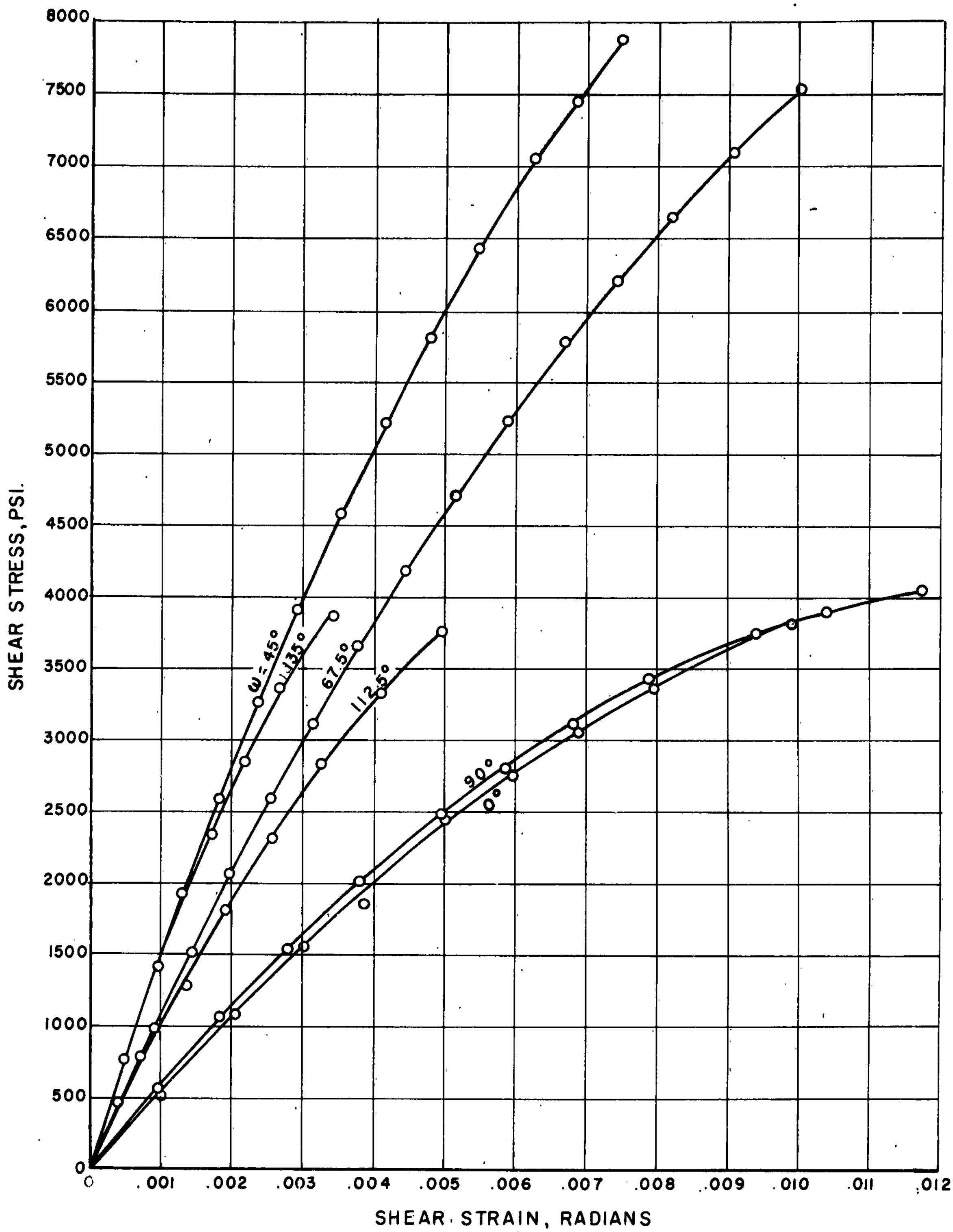
DESIGN DATA SHEET

FIGURE I



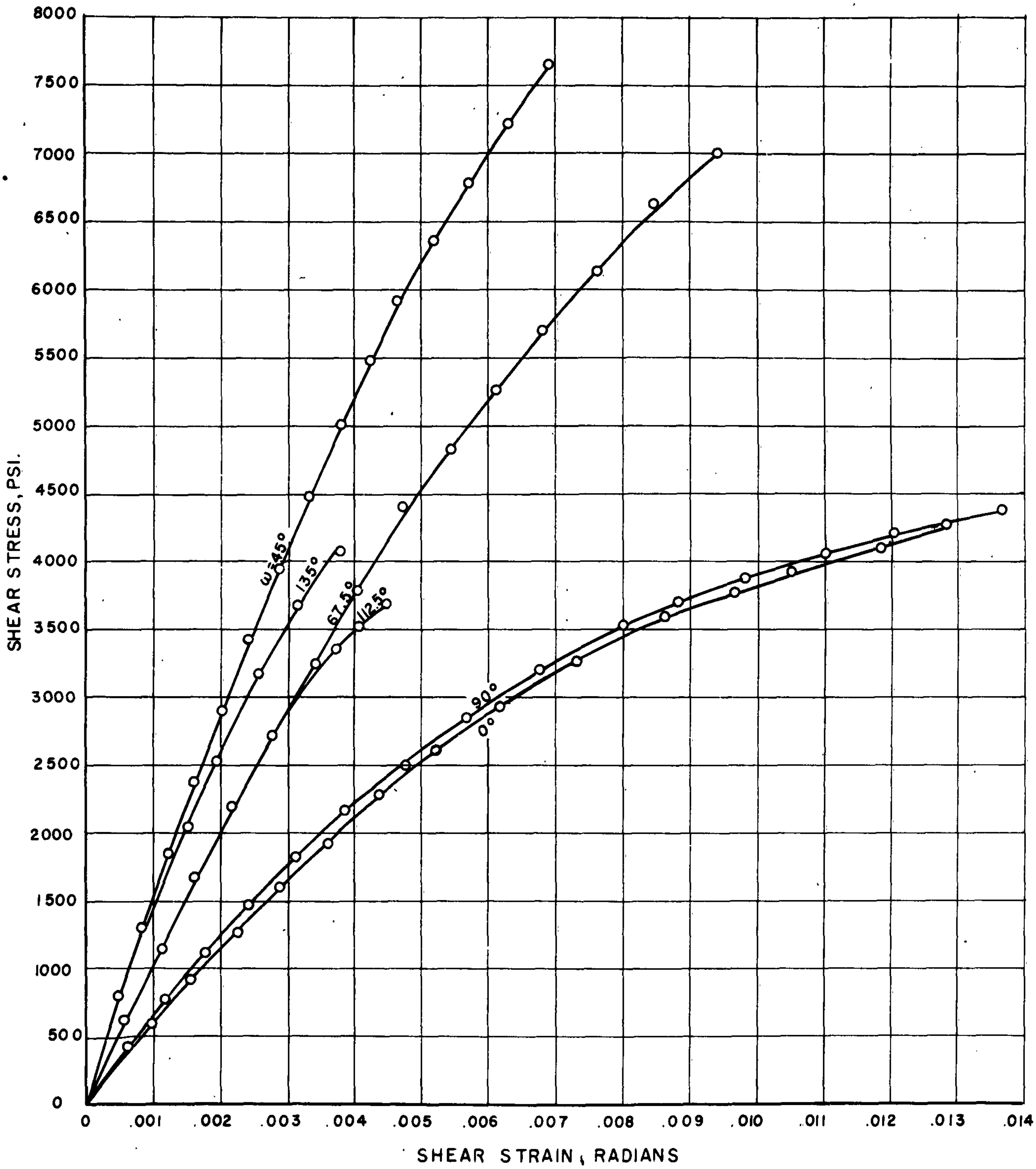
SHEAR PROPERTIES OF WOVEN ROVING LAMINATE

FIGURE 2



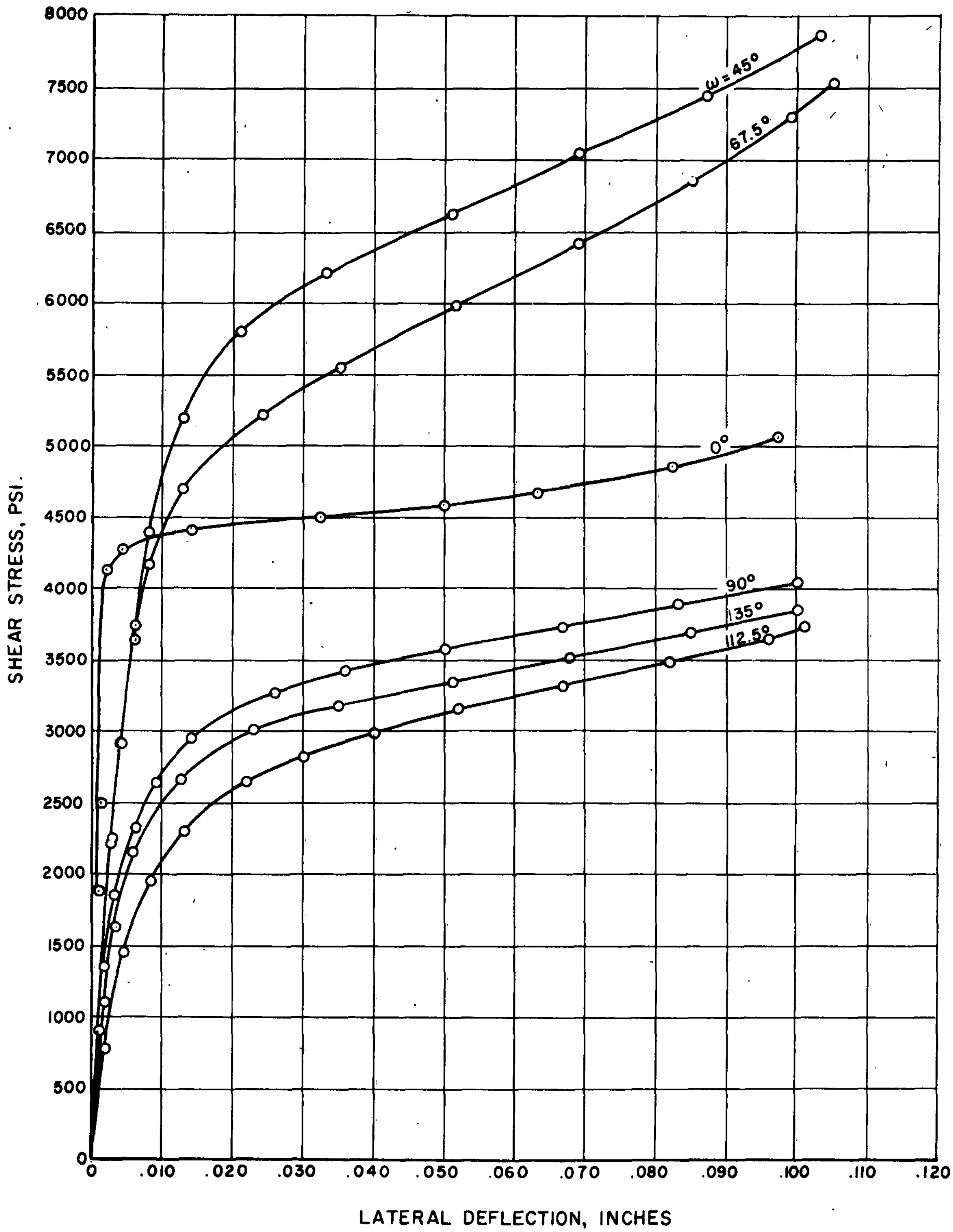
SHEAR STRESS-STRAIN CURVES FOR VARIOUS ANGLES, ω
WOVEN ROVING LAMINATE, SHEET I

FIGURE 3



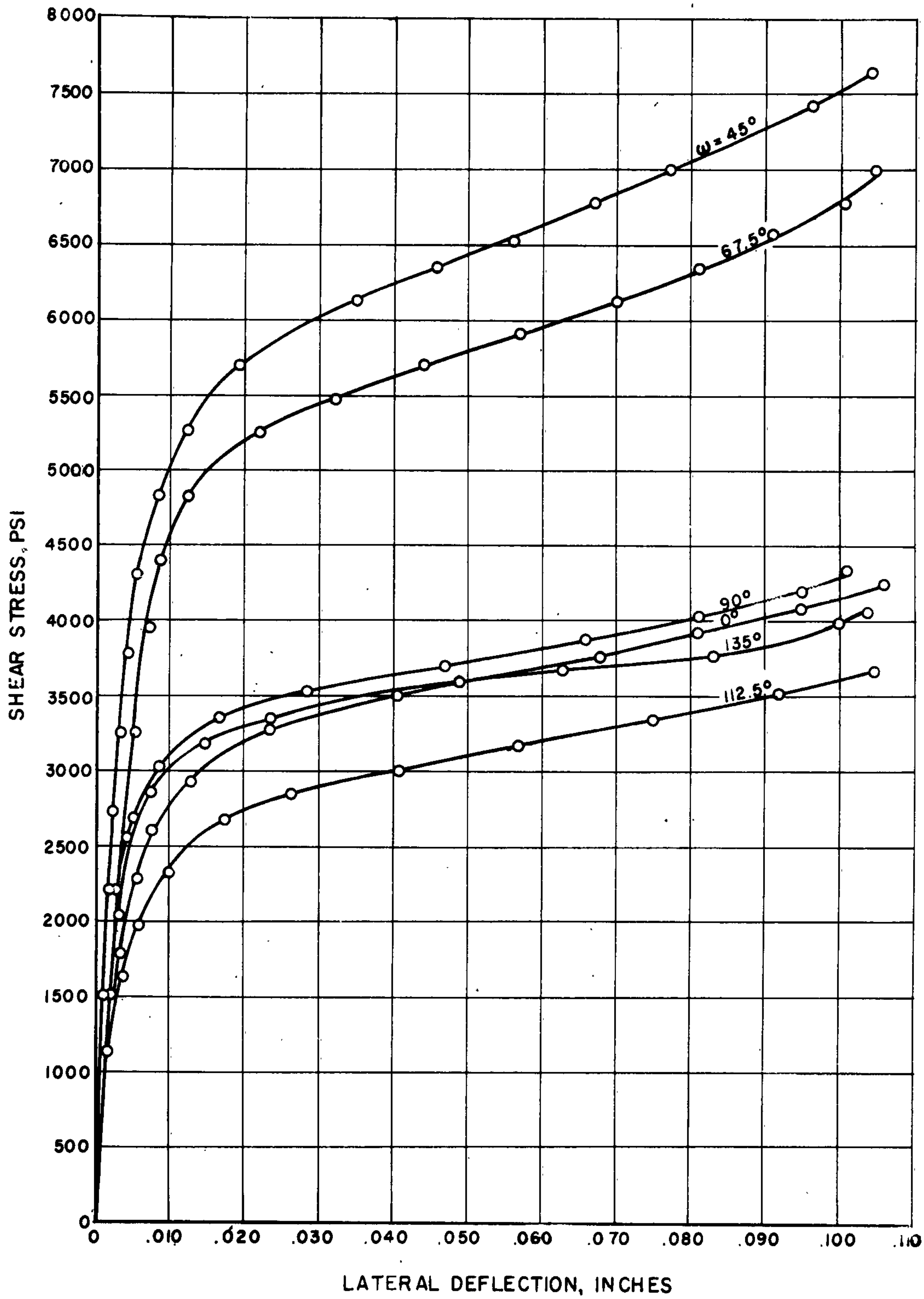
SHEAR STRESS-STRAIN CURVES FOR VARIOUS ANGLES ω
WOVEN ROVING LAMINATE, SHEET 2

FIGURE 4



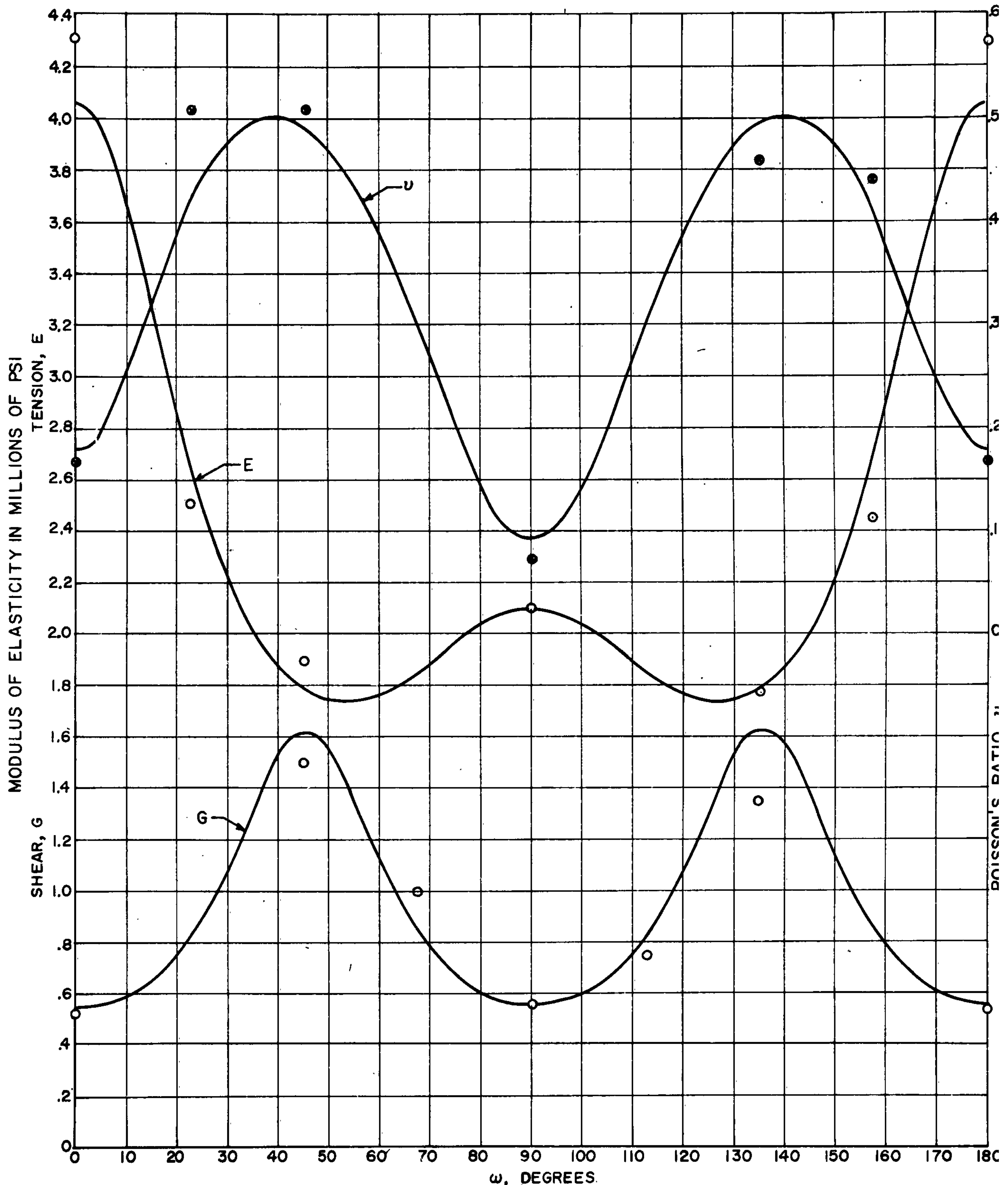
SHEAR BUCKLING OF WOVEN ROVING LAMINATE, SHEET I
8x8 in PANELS LOADED AT VARIOUS ANGLES, ω
NOMINAL THICKNESS = 0.1 in

FIGURE 5



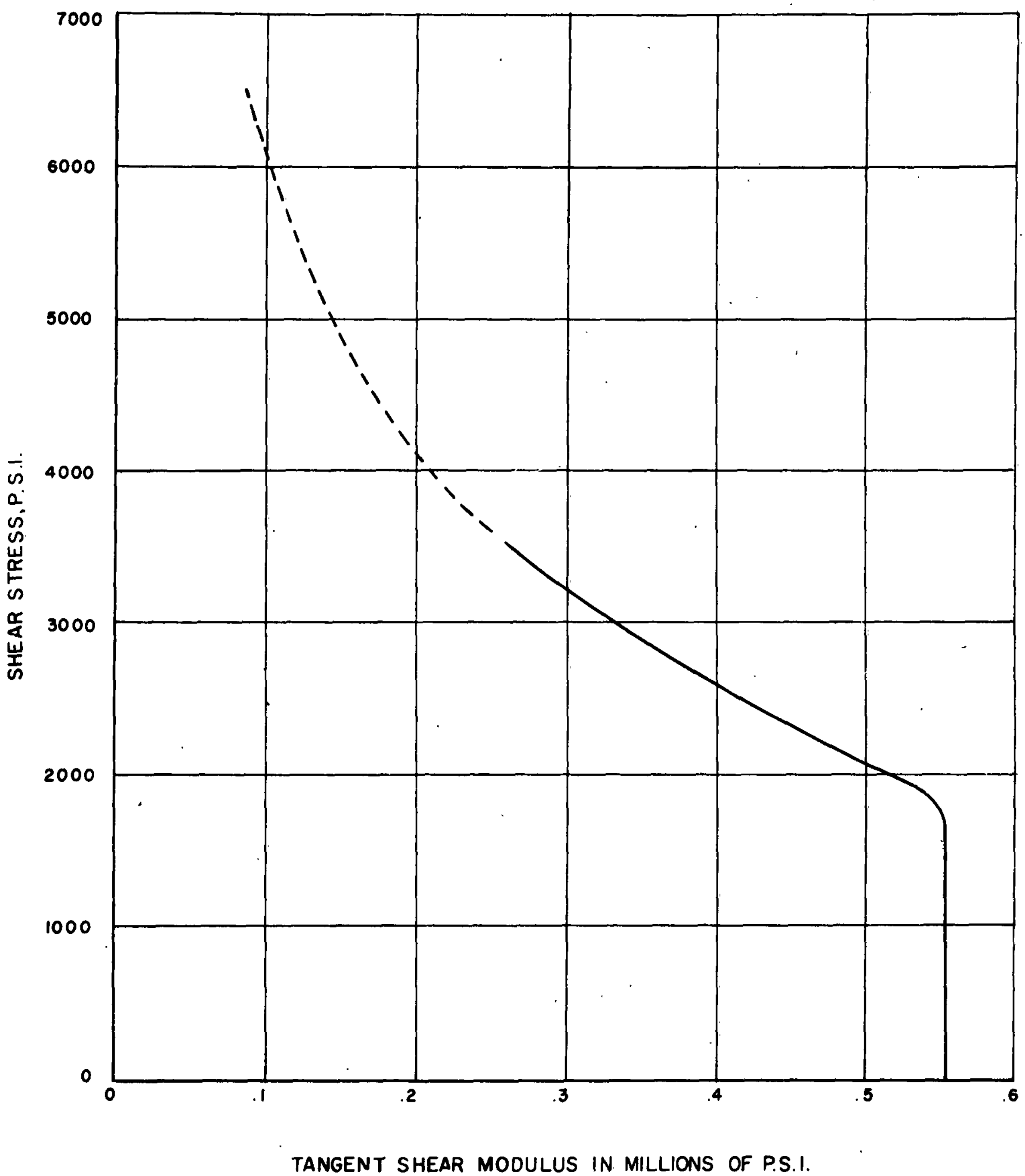
SHEAR BUCKLING OF WOVEN ROVING LAMINATE, SHEET 2
8×8 in PANELS LOADED AT VARIOUS ANGLES, ω
NOMINAL THICKNESS = 0.1 in

FIGURE 6



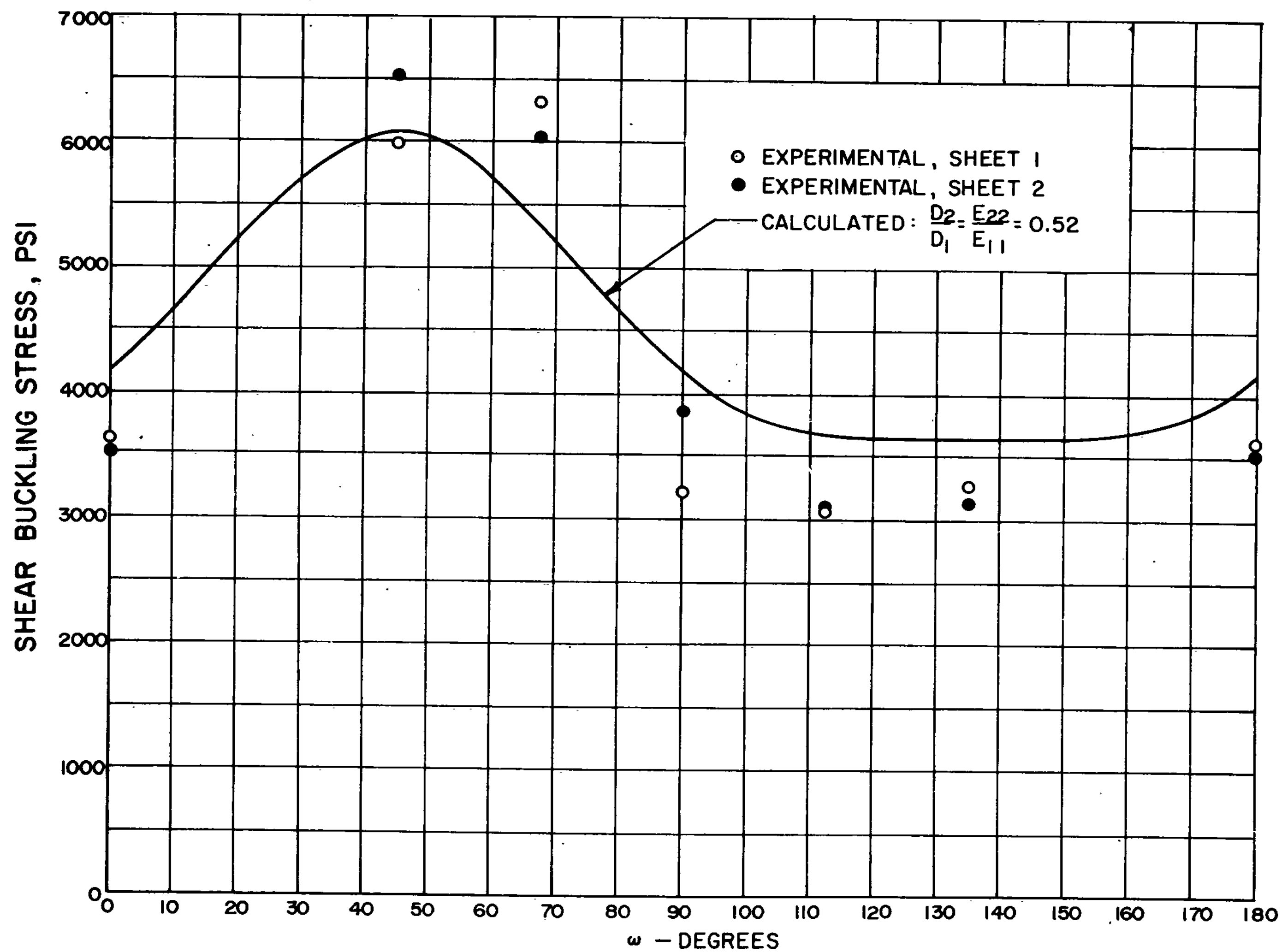
MECHANICAL PROPERTIES FOR VARIOUS ANGLES, ω
WOVEN ROVING LAMINATE, SHEET I

FIGURE 7



VARIATION OF TANGENT SHEAR MODULUS WITH SHEAR STRESS
WOVEN ROVING LAMINATE $\omega = 90^{\circ}$

FIGURE 8



SHEAR BUCKLING OF WOVEN ROVING LAMINATE
8x8x0.1 IN. PANELS WITH CLAMPED EDGES

FIGURE 9

TABLE 1

MECHANICAL PROPERTIES OF WOVEN ROVING LAMINATE

Specific Gravity	Glass Content Percent by Weight	Loading Direction ω Degrees	Modulus of Elasticity Millions of P.S.I.			Poisson's Ratio ν	Ultimate Tensile Strength P.S.I.
			Tension	Compression	Flexure		
1.81	65.8	0	4.31	3.83	3.70	0.17	69,700
		22.5	2.50	2.61	2.55	0.51	15,500
		45.0	1.89	1.70	1.51	0.51	11,300
		90.0	2.09	2.17	1.67	0.07	27,700
		135.0	1.76	1.78	1.34	0.46	11,400
		157.5	2.46	2.35	2.86	0.39	17,300

NOTE: 1. Laminate was fabricated from 5 plies of 205-50 Cloth and #8000 Polylite resin press cured at 150 to 250°F to 1/8 in stops.

2. Each value tabulated represents an average of 3 results.

TABLE 2

MODULUS OF ELASTICITY IN SHEAR, G, FOR VARIOUS ANGLES ω -SHEETS 1 AND 2

Angle ω Degrees	Modulus of Elasticity in Shear, G Millions of P.S.I.	
	Sheet 1	Sheet 2
0	0.533	0.558
45	1.500	1.438
67.5	1.015	1.000
90	0.540	0.625
112.5	0.930	0.978
135	1.330	1.375

TABLE 3

CRITICAL BUCKLING SHEAR STRESSES τ_{cr}
WOVEN ROVING LAMINATE

ω Degrees	Sheet No.	Thick-ness t Inch	Experimental τ_{cr} psi Actual	Corrected to t=0.1 in. Actual	Corrected to t=0.1 in. Average	Calculated τ_{cr} psi t=0.10 in.	Ratio Exp./Calc.
0	1	0.113	4600	3600	3570	4180	0.86
	2	0.102	3600	3530			
45	1	0.105	6600	5990	6270	6100	1.03
	2	0.099	6410	6540			
67.5	1	0.097	5940	6310	6180	5340	1.16
	2	0.098	5800	6040			
90	1	0.106	3600	3210	3550	4180	0.85
	2	0.098	3730	3880			
112.5	1	0.101	3130	3070	3090	3690	0.84
	2	0.100	3100	3100			
135	1	0.101	3340	3280	3210	3650	0.88
	2	0.107	3580	3130			

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